Pricing and Hedging Jamaican Dollar Quanto Options on the S&P 500 Index

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Abstract

This paper presents a practical framework for pricing and hedging Jamaican Dollar (JMD) denominated quanto options, also known as guaranteed exchange rate options, on the S&P 500 Index. Historically, low liquidity in domestic equities has constrained the availability of advanced derivative products for local dealers. By leveraging the deep liquidity of the USDJMD spot market and US-listed options, this study demonstrates how local dealers can synthetically manufacture and dynamically hedge these exotic instruments. The proposed self-financing structure successfully isolates market exposure from exchange rate volatility. We also show that dynamic hedging strategies that incorporate adjusted delta positions and dynamic USD spot trading effectively mitigate the dealer's residual market and FX risks. Ultimately, this framework provides a scalable foundation for expanding the universe of JMD-investable securities and advancing local capital market capabilities.

1 Background and Motivation

Relatively low levels of liquidity in Jamaica's equities market presents a problem for dealers when trying to hedge an option written on a Jamaican stock. Therefore, options related transactions may only occur when there are two counterparties with opposing views whereas in the US, options market makers hedge their net positions by trading in the underlying stock.

The goal of this paper is to demonstrate the viability of providing to Jamaican investors, institutional or retail, access to a liquid options market without exchange risk. In this case, the dealer of this security takes on the exchange rate risk and hedges that risk by trading in the spot FX market daily. The structure is self-financing, meaning the dealer can incorporate borrowing costs associated with hedging in the price quoted to a counterparty in a similar way forwards are priced to incorporate interest rate differentials.

This structure has the potential to expand the universe of tradable Jamaican dollar securities and investment strategies. Investors could:

- Bet on US market volatility.
- Get exposure to a large number of securities to diversify their equities portfolio.
- Expand wealth offerings to include market linked notes to their clients.

All of this could be done without taking on the FX risk associated with buying foreign securities. This structure applies to any market so long as the FX market, the options market and stock markets are liquid. The idea also expands to futures and forwards as well such as US short term Interest rate futures and commodity futures. The focus of this paper however is on US Index etf options.

Asset Class	Dec 2022 JMD' B	Mar 2023 JMD' B	Jun 2023 JMD' B	Sep 2023 JMD' B	Dec 2023 JMD' B	Quarter over Quarter Dec 23 percentage change	Year over Year Dec 23 percentage change
(a) Real Estate	32.10	31.18	35.75	36.15	34.32	-5.07	6.91
(b) Equities	99.40	93.90	90.23	87.59	87.22	-0.43	-12.26
(c) Fixed Inc. Securities	186.70	201.37	208.70	199.08	193.32	-2.89	3.55
(d) Cash	8.90	7.15	6.91	6.30	11.81	87.49	32.67
(e) Other assets	19.70	11.26	10.40	16.96	21.93	29.33	11.33
Total	**346.80**	**344.85**	**351.99**	**346.07**[5]	**348.60**	**0.73**	**0.52**

Figure 1: Table showing size and Portfolio Composition of the Jamaican CIS Industry taken from the Financial services Commission December 2023 Quarterly Summary

As at December 2023 the Jamaican collective investment schemes had roughly 87 Billion JMD allocated to equities.

The USDJMD FX spot market is relatively liquid with a daily average of roughly 50 Million USD from November 2023 to November 2024. The SPX Index options and SPY etf options have a average daily volume of 9 Million Contracts traded Daily as well as the SPY ETF has a average daily volume traded of 40 Million.

2 Pricing

The Guaranteed exchange rate option, more commonly known as the Quantity-Adjusting or Quanto Option, is an exotic option that gives an investor exposure to a foreign asset without taking on FX risk as the exchange rate is guaranteed throughout the life of the option. As the name suggests the quantity of foreign currency exposure hedging embedded in the option is changing as the difference between the option's strike and the underlying spot price changes.

The payoff of a quanto option on the foreign stock index at expiration is:

$$C = \max[X_0(S_T - K), 0]$$

Where X_0 is the guaranteed rate agreed at the beginning of the contract, S_T is the index level at the time of expiration and K is the option's strike. Given that the index we are looking at in this paper has the deepest most liquid options market in the world, instead of a model approach to pricing we can simply apply the guaranteed rate to any option contract that is being traded publicly. Therefore we can think of the daily price quoted to a client by a dealer of such a contract as the following:

$$C_{JMD} = C_{USD} X_0$$

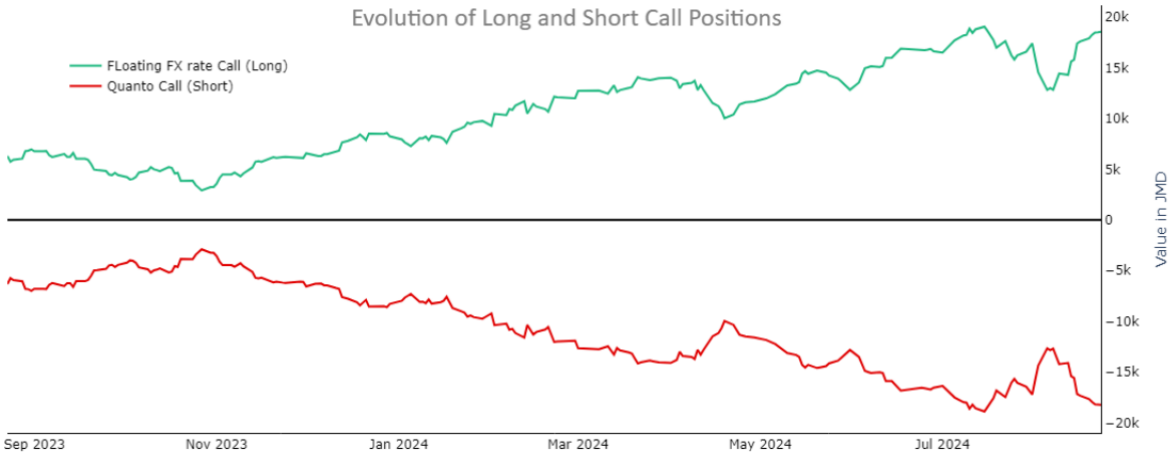
Where, same as before, X_0 is the guaranteed rate and C_{USD} is a USD denominated Call option traded on a public exchange in the United States. This removes the need to delta hedge in the underlying security as an identical option can be purchased after the premium received in Jamaican dollars is converted to US dollars at the prevailing FX rate. This approach however still leaves the dealer with FX exposure which can be hedged dynamically. While this introduces the need to consider the interest rate differential due to the need to sell short USD to hedge the FX risk, for the sake of this paper we will assume the funding rate for both currencies are the same[1].

[1]When hedging FX exposure the dealer will need to borrow USD and sell short after which they will then place the JMD received in an interest bearing account for the necessary period. A difference in these rates must be reflected in the price quoted to a client to avoid arbitrage opportunities

3 Hedging

When a dealer sells a call option denominated in JMD with no guaranteed exchange rate the dealer has left this currency risk with the counterparty. With a guaranteed exchange rate however, the dealer has taken on that risk and now needs to hedge that as well. To better understand this, we can think of the quanto option essentially as a portfolio of securities. A written (sold) call can be replicated with the dealer first buying USD with the JMD premium received then using that USD to buy a call option with the same underlying, strike and expiry. When the exchange rate is close to the guaranteed rate this portfolio will behave like to a long non-deliverable forward with the guaranteed exchange rate quoted to the counterparty as the forward rate. As the exchange rate moves further from the guaranteed rate however additional positions are needed to hedge those movements.

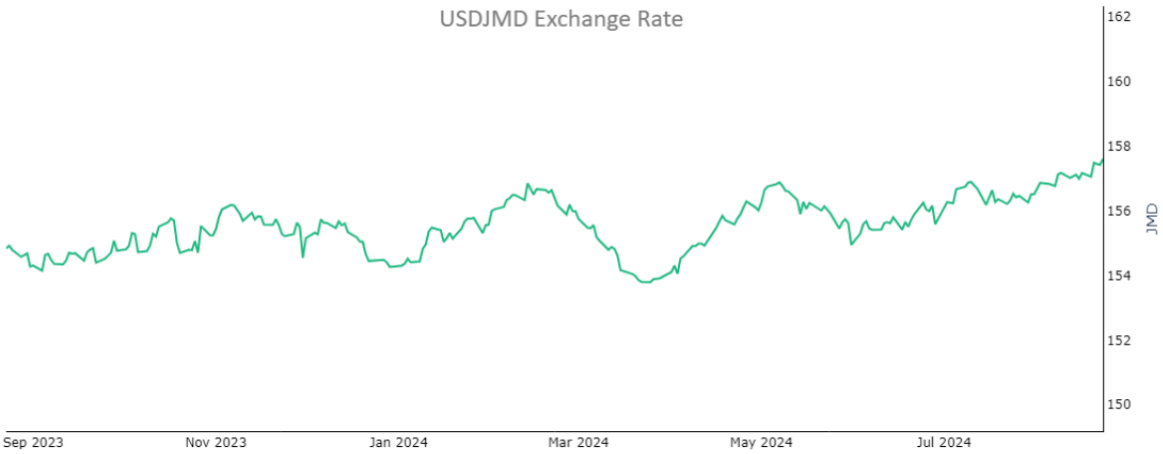
Below is a graph of the evolution of two components of the a hypothetical dealer portfolio; a Long position (green) in a SPY[2] etf call option with a strike price of 445 written in August 23, 2023, a point in time when the SPY etf closed at $443.03 and short position (red) in an identical 445 call traded on an options exchange in the U.S. The Long position is converted to Jamaican dollars at the prevailing fx rate while the short position is denominated in Jamaican dollars at a fixed exchange rate of 154.84[3].



The long position being denominated in Jamaican dollars exposes the dealer to the USDJMD exchange rate fluctuations which is the main source of market risk in the portfolio when the exchange rate is close to the guaranteed rate as the spy etf exposure is mostly hedged with two opposite and identical payoffs. Once the option can be statistically hedged the exchange rate becomes the sole source of market risk.

[2]The historical data for the SPDR S&P 500 etf call option was used as the etf as well as its options market are very liquid and has a very small tracking error to the S&P 500 index

[3]The implication of the funding rate in both currencies being equal is that the forward rate is equivalent to spot which is illustrated by our scenario as the guaranteed rate of 154.84 was the USDJMD spot rate on august 23,2023.



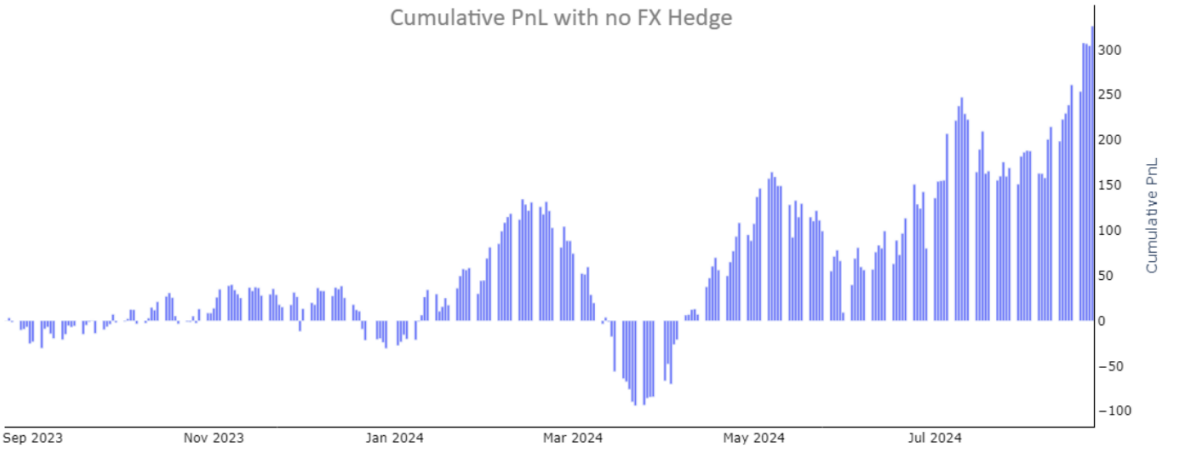
Over the period the Jamaican dollar weakened against the USD causing the value of the long floating exchange rate call position to gain value relative to the short fixed exchange rate call position. Therefore the PnL becomes increasingly positive as the exchange rate increases. While this is favorable for the dealer, had the opposite occurred, if the Jamaican dollar had strengthened against the USD, the PnL would have been negative, meaning the long floating exchange rate call position would have lost value relative to the short fixed exchange rate call position.

The portfolio without FX hedge can be summarized as follows:

$$\text{PnL} = -C_{USD}X_0 + C_{USD}X_{JMD}$$

Where:

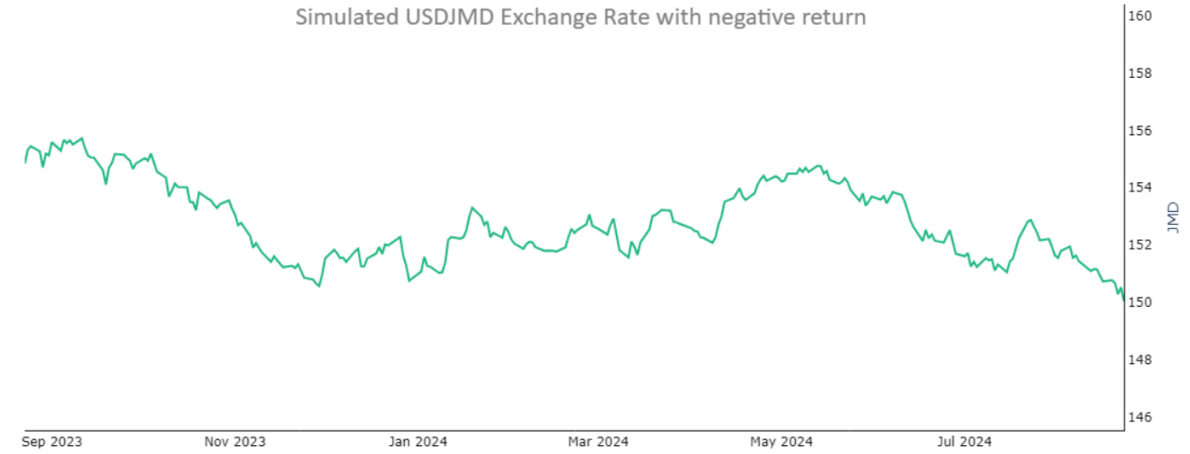
- C_{USD} : USD call option traded in the Market
- X_{JMD} : Prevailing USDJMD Exchange Rate
- X_0 : Guaranteed Exchange Rate

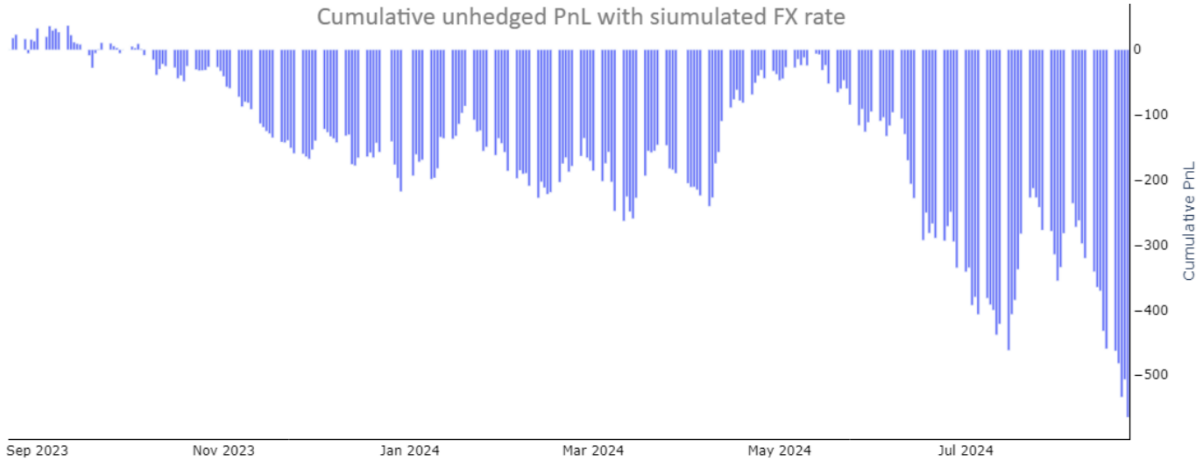


A Monte Carlo path of the USDJMD exchange was generated with a hypothetical negative return[4]. A similar portfolio was then constructed with the same call option and guaranteed

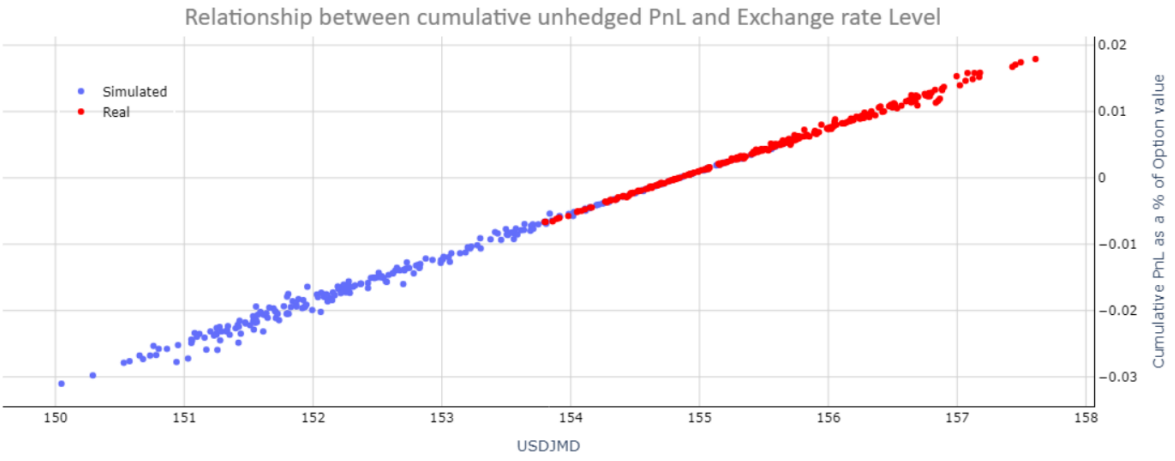
[4]A geometric brownian motion framework with a mean of -1% and a standard deviation of 3% was used to run the simulation as the price paths of this equation are continuous and visually similar to the USDJMD Exchange rate.

exchange rate with the simulated exchange rate shown below in which the cumulative PnL was also correlated to the evolution of the spot exchange rate.





Combining both data sets we can look at the relationship between the cumulative PnL of our portfolio without FX hedging as a percentage of the option contract value and the floating exchange rate level. As seen below the relationship is essentially the payoff diagram of a non-deliverable forward around the 154.84 level showing that as the exchange rate gets closer to the guaranteed rate the cumulative PnL of both portfolios approach zero.



This exposure can be hedged by borrowing USD and selling short in the spot market. The quantity of USD required to hedge the daily fluctuations is equivalent to the USD price of the option. For example, the option's price on August 23, 2023 was US$40.78. A dealer will need to borrow this amount and exchange it for JMD. On August 24, 2023 the price of the option was trading at US$37.04. The dealer already being short US$40.78 will need to buy back US$3.74 reducing their short position to what the current USD exposure is.

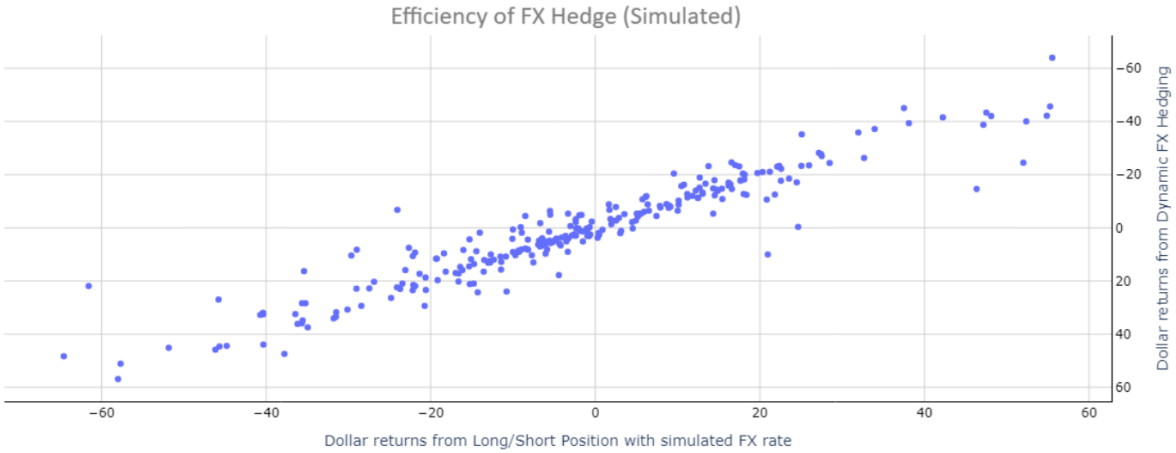
The portfolio with FX hedge can be summarized as follows:

$$\text{PnL} = -C_{USD}X_0 + C_{USD}X_{JMD} - C_{X_{JMD}}$$

Where:

- $-C_{X_{USDJMD}}$: Short USD Position equivalent to the size of long call option

This short USD position is necessary but not sufficient to neutralize the movement of the long call position in our portfolio and will continue throughout the life of the option.



While our simulated exchange rate had a higher volatility than the real exchange rate we can see that the relationship between the fx exposure and the offsetting positions remains the same. This is useful as losses can compound and having offsetting returns on a daily basis can reduce the likelihood of losses to the dealer at expiration significantly.

So far we have largely ignored the co-movement between the underlying and the exchange rate. As mentioned before, when the current exchange rate is close to the guaranteed rate our long call and short USD positions is enough to hedge the risk of our short call position. However

as the exchange rate moves further from the guaranteed rate we must account for this with an additional term as each daily change between our long floating exchange rate call exposure and our short fixed exchange rate call is exposure is amplified by the difference in the two rates. This exposure can actually be summarized by altering the short call's delta[5] by $\frac{X_0}{X_{JMD}}$.

Replacing our long call position with this adjusted delta, our portfolio now looks like this:

$$\Pi = -C_{USD}X_0 + \eta_S - \eta_S USD$$

Where:

$$\eta_S = \Delta_{BS}\frac{X_0}{X_{JMD}}S_{USD}$$

- Δ_{BS} : Black-Scholes delta of the call option
- S_{USD} : Underlying denominated in USD
- $\eta_S USD$: Total Amount of USD needed to hedge FX exposure

As we have adjusted how much USD exposure we are now taking we must also adjust how much USD we are short hence we have replaced $C_{X_{JMD}}$ with $\eta_S USD$. Also, if we assume $C_{USD} = \Delta_{BS}S_{USD}$ we can simply subtract 1 from our adjusted delta position and keep the long call position in our portfolio with our new FX hedge and an additional delta position which can be summarized as follows:

$$\Pi = -C_{USD}X_0 + C_{USD}X_{JMD} + \zeta - \zeta_{USD}$$

$$\zeta_{USD} = C_{USD} + \zeta$$

$$\zeta = \Delta_{BS}\left(\frac{X_0}{X_{JMD}} - 1\right)S_{USD}$$

This is more practical as one can buy a fractional shares with most US brokers however we cant buy fractional options. This distinction however is irrelevant for a dealer who's book is hundreds of millions of dollars.

Summarized below is the daily average portfolio change with all the components used to hedge our short quanto option.

	Actual Exchange Rate		**Simulated Exchange Rate**	
	Mean Daily PNL (JMD)	**Standard Deviation (JMD)**	**Mean Daily PNL (JMD)**	**Standard Deviation (JMD)**
II With Long call and FX Hedge Only	0.45	3.86	-0.91	7.15
II With Long call, FX Hedge and Additional Delta position	-0.06	2.70	-0.04	5.07

Table 1: Comparison of PNL under Actual and Simulated Exchange Rates

[5]Assuming the correlation between the underlying and the exchange rate is not zero, the delta would be adjusted to incorporate a dividend yield:

$$q' = q + \rho_{xy}\sigma_x\sigma_y$$

where q is the dividend yield, ρ_{xy} is the correlation between the index (x) and the exchange rate (y), σ_x is the volatility of the index, and σ_y is the volatility of the exchange rate. The 5-year correlation between the SPY and the USD/JMD exchange rate was -0.02 and, for the sake of simplicity, is assumed to be zero.

Adding our additional delta position significantly reduces the average daily change in our portfolio as well as the standard deviation in both exchange rate evolutions. Summarized below is the final portfolio PNL comparing just our long call position with the addition of our FX hedge and our additional delta position.

	Actual Exchange Rate		
	PnL without FX Hedge	**PnL with FX Hedge**	**PnL with FX Hedge and Additional Delta Position**
in Dollars (JMD)	326.08	113.41	-20.88
as % of Initial Contract Price	5.16%	1.80%	-0.33%
as % of Initial Notional Amount	0.48%	0.17%	-0.03%

Table 2: Final Portfolio PNL (Actual Exchange Rate)

	Simulated Exchange Rate		
	PnL without FX Hedge	**PnL with FX Hedge**	**PnL with FX Hedge and Additional Delta Position**
in Dollars (JMD)	-564.89	-228.66	-4.21
as % of Initial Contract Price	-8.95%	-3.62%	-0.07%
as % of Initial Notional Amount	-0.82%	-0.33%	-0.01%

Table 3: Final Portfolio PNL (Simulated Exchange Rate)

The main takeaway here is that each component of our portfolio hedges a specific risk:

- Long Call + Adjusted Delta : Hedges short quanto payoff risk
- Short USD : shorts USDJMD exposure of the Long Call + Adjusted delta position

4 Conclusion

US listed Options make the manufacturing of quanto options simpler for a Jamaican dealer as hedging the risks associated takes place is two fairly deep and liquid markets, the USDJMD spot market and the US SPY ETF options market. Providing quanto options to Jamaican investors will require institutional access to both markets to reduce the effects of commissions and bid-ask spreads as these will need to be transferred to the client and wide spreads can discourage trading as it negatively impacts returns.

References

[1] Demeterfi, Kresimir. "How to Value and Hedge Options on Foreign Indexes." *Goldman Sachs Quantitative Strategies Research Notes*, 1998.

[2] Financial Services Commission. "Securities Industry December 2023 Quarterly Summary" September 2024.